

Scattering by a free electron (Thomson-scattering)

And because in a plane wave

$$E = E_0 e^{-i(\omega t - K \cdot r)}$$

eqn (4) becomes

$$\frac{d^2x}{dt^2} = \frac{q/E_0}{m} e^{-i(\omega t - K \cdot r)} \quad (5)$$

Equation (5) implies that the acceleration, velocity and displacement of the particle are all in the same direction as E_0 which itself is constant and that the charge is oscillating sinusoidally.

Now if the incident electromagnetic wave in which the electric vector is along x -axis (as it is polarised) is moving along x -axis as shown in fig. then the acceleration in the x -direction will be given by

$$\frac{d^2x}{dt^2} = \frac{q/E_0}{m} e^{-i(\omega t - kx)} \quad (7)$$

so that the displacement x at time t will be given by

$$x = \frac{q/E_0}{m\omega^2} e^{-i(\omega t - kz)}$$

Now as an oscillating behaves like an oscillating dipole with dipole moment

$$\vec{p} = q\vec{x}$$

It follows from eqn (7) that

$$p = \frac{q^2 E_0}{m \omega^2} e^{-i(\omega t - kx)}$$

or $p_0 = \frac{q^2 E_0}{m \omega^2}$ --- (8)

But as the average energy radiated per sec per unit area in a normal direction by an oscillating dipole is given by

$$S_{sr} = \frac{1}{4\pi \epsilon_0} \frac{\omega^4 p_0^2}{8\pi c^2 r^2} \sin^2 \theta$$

or $S_{sr} = \frac{1}{4\pi \epsilon_0} \frac{\omega^4}{8\pi c^2 r^2} \left[\frac{q^2 E_0}{m \omega^2} \right] \sin^2 \theta$

(putting the value of p_0 from eqn 8)

i.e. $S_{sr} = \frac{1}{4\pi \epsilon_0} \frac{q^2 E_0^2}{8\pi m^2 c^2 r^2} \sin^2 \theta$ --- (9)

Further as for a plane wave

$$S_{ir} = E \times H$$

So the average value of S_{ir} will be given

by

$$S_{ir} = \frac{1}{2} E_0 H_0 \quad (\text{as } E \text{ is } \perp H)$$

i.e. $S_{ir} = \frac{1}{2} \epsilon_0 c E_0^2 \quad (\text{as } H_0 = c \epsilon_0 E_0)$ --- (10)

So the ~~average~~ differential scattering cross-section

$$\frac{d\sigma}{d\Omega} = \frac{S_{sr} r^2}{S_{ir}}$$

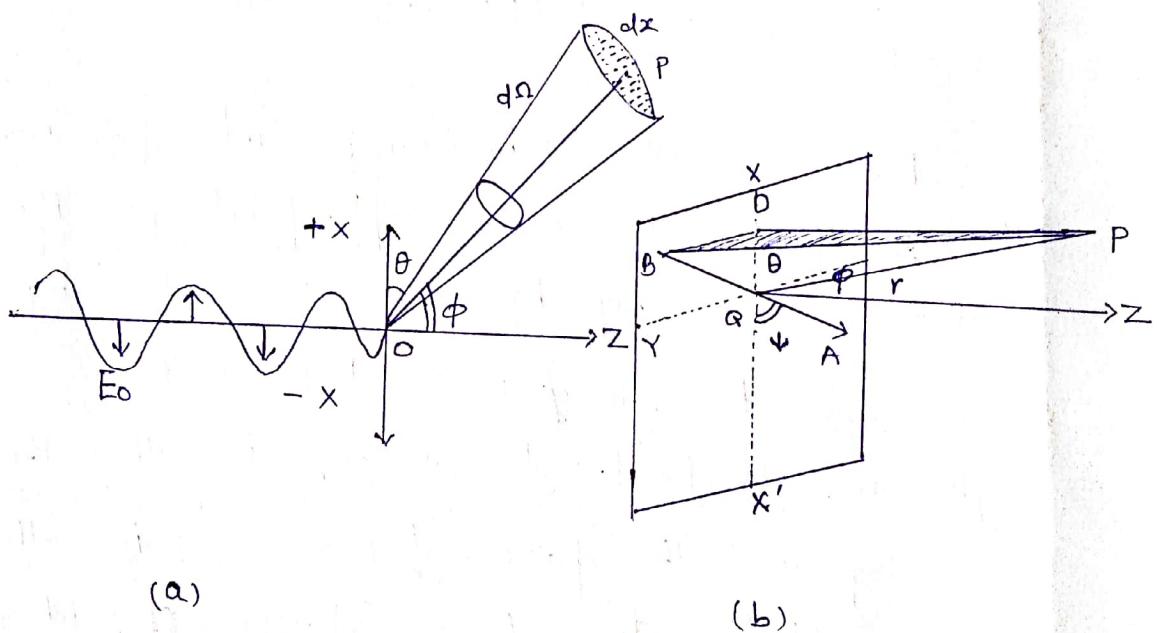
$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q^4 E_0^2 \sin^2 \theta}{8\pi m^2 c^2} \times \frac{2}{\epsilon_0 c E_0^2}$$

$$= \left(\frac{q^2}{4\pi\epsilon_0 m c^2} \right)^2 \sin^2 \theta$$

or $\frac{d\sigma}{d\Omega} = \gamma_0^2 \sin^2 \theta = \gamma_0^2 \cos^2 \phi \quad \dots \dots \dots \quad (A)$

where $\gamma_0 = (q^2/4\pi\epsilon_0 m c^2)$ and is called the classical radius of the electron.

The incident radiation has been taken to be plane polarised. For unpolarised waves an average must be taken over all orientations of the plane of E .



Suppose in figure above AB is the direction of E in another wave incident on the particles of figure (a). So that the angle between AB and the plane of fig. (a) containing field point ϕ . It is

now preferable to express the scattering in the terms of angle ϕ which is common to all azimuths. In fig.

(b) the plane POB is drawn perpendicular to the plane containing AB so that the length BQ is given both by $r \cos \theta$ and by $r \sin \phi \cos \psi$.

$$\text{i.e. } \cos^2 \theta = \sin^2 \phi \cos^2 \psi$$

$$\text{i.e. } \sin^2 \theta = 1 - \cos^2 \psi (1 - \cos^2 \phi)^* \quad \dots \dots \dots \quad (11)$$

$$\sin^2 \theta = 1 - \frac{1}{2} (1 - \cos^2 \phi) \quad [\text{as } (\cos^2 \psi)_{av} = \frac{1}{2}] \quad \dots \dots \dots \quad (12)$$

Substituting the value of $\sin^2 \theta$ from eqn (12) in (A), we get

$$\frac{d\sigma}{d\Omega} = r_0^2 \frac{1}{2} (1 + \cos^2 \phi) \quad \dots \dots \dots \quad (B)$$

This is called Thomson formula for scattering of radiation and is appropriate for the scattering of X-rays by electron or γ -ray by proton. In it angle ϕ is called the scattering angle and the factor $\frac{1}{2} (1 + \cos^2 \phi)$ is called the scattering and the factor $\frac{1}{2} (1 + \cos^2 \phi)$ is called degree of dipolarisation. From expression (B) it is clear that

(i) scattering of electromagnetic waves is independent of that nature of incident wave (i.e. ω).

Continue

(30-09-2020)